ASSIGNMENT SET – I

Mathematics: Semester-II

M.Sc (CBCS)

Department of Mathematics

Mugberia Gangadhar Mahavidyalaya

PAPER - MTM-206

Paper: General Topology

1. A topological space X is compact if and only if for every collection C of closed sets in X having finite intersection property.
2. Show that every path connected space is a connected space .Is converse also true? Explain.
3. Examine the compactness of the following sets over the interval $(0,1)$
   1. $\left\{\left(sin^2(\frac{n\pi}{100}), cos^2\left(\frac{n\pi}{100}\right)\right): n \in \mathbb{N}\right\}$
   2. $\left\{\left(\frac{1}{2}e^{-\pi}, 1-\frac{1}{(n+1)^2}\right): n \in \mathbb{N}\right\}$

4. Let X and Y be two topological space ,$f: X \to Y$be a mapping then following are equivalent

(a)f is continuous

(b)for every closed set B of Y the set $f^{-1}(B)$ is closed in X.

5. Let two topologies τ_1 and τ_2 on a non empty set X and if β_1 and β_2 are two basis of τ_1 and τ_2 respectively. Then following are equivalent…..

(i)$\tau_1 \subset \tau_2$

(ii)For every $x \in B_1, B_1 \epsilon \beta_1 \exists$ elementB_2 of $\beta_2$such that $x \in B_2 \subset B_1$

6. Let X be a metrizable topological space. Show that following are equivalent

(a) Every continuous function $f: X \to \mathbb{R}$ is bounded .
(b) X is limit point compact.

7. Let A be a connected subspace of X If $A \subset B \subset \overline{A}$ then B is also connected.

8. Show that k-th topology is finer than the standard topology**.**

9. Define quotient topology

10. Show that every compact Hausdorff space is normal space.

11. Consider the set N with the family Ʈ of its subset consisting ϕ and all subsets of the form

$A_k = \{k, k+1, k+2, \ldots \ldots.\} where\ k = 1,2,3 \ldots$

show that Ʈ is a topology.

12. Discuss the connectedness of the following sets-

A. $\left\{x \sin\frac{1}{x} : x \in (0,1)\right\}$

B. $\{|x| : x \in (-1,1)\} \cap \{e^x : x \in R\}$

13. Give an example of which (X_1, τ_1) is T_3 space and τ_1 is subset of τ_2 but (X_2, τ_2) is not T_3 space.

14. Let $\{A_\alpha\}$ be a collection of subsets of X .Let $X = \bigcup A_\alpha$ and f:X→ Y .Let f restricted to A_α is continuous for each α.

(a) Show that if the collection is finite and if each A_α is closed then f is continuous.

(b) Find the example where the collection A_α is infinite and each A_α is closed but f is not continuous .

15. Show that every T_1 space is T_0 space but the converse is not true in general (give example).

16. Show that X is Hausdorff space iff the diagonal $\Delta = \{(x, x): x \in X\}$ is closed in $X \times X$.

17. If a topological space is T_2 space then every convergence sequence has a unique limit. But this may not true in genera (give example)

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